4.3.1 cont...

(d) Freq sampling form

Let H[k] - N point DFT of N point sequence h[n]

$$H(z) = \sum_{n=0}^{N-1} h[n] z^{-n}$$

$$= \sum_{n=0}^{N-1} \left\{ \text{IDFT of } H[k] \right\} z^{-n}$$

$$= \sum_{n=0}^{N-1} \left\{ \frac{1}{N} \sum_{k=0}^{N-1} H[k] W_N^{-kn} \right\} z^{-n}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} H[k] \left(W_N^{-k} z^{-1} \right)^n$$

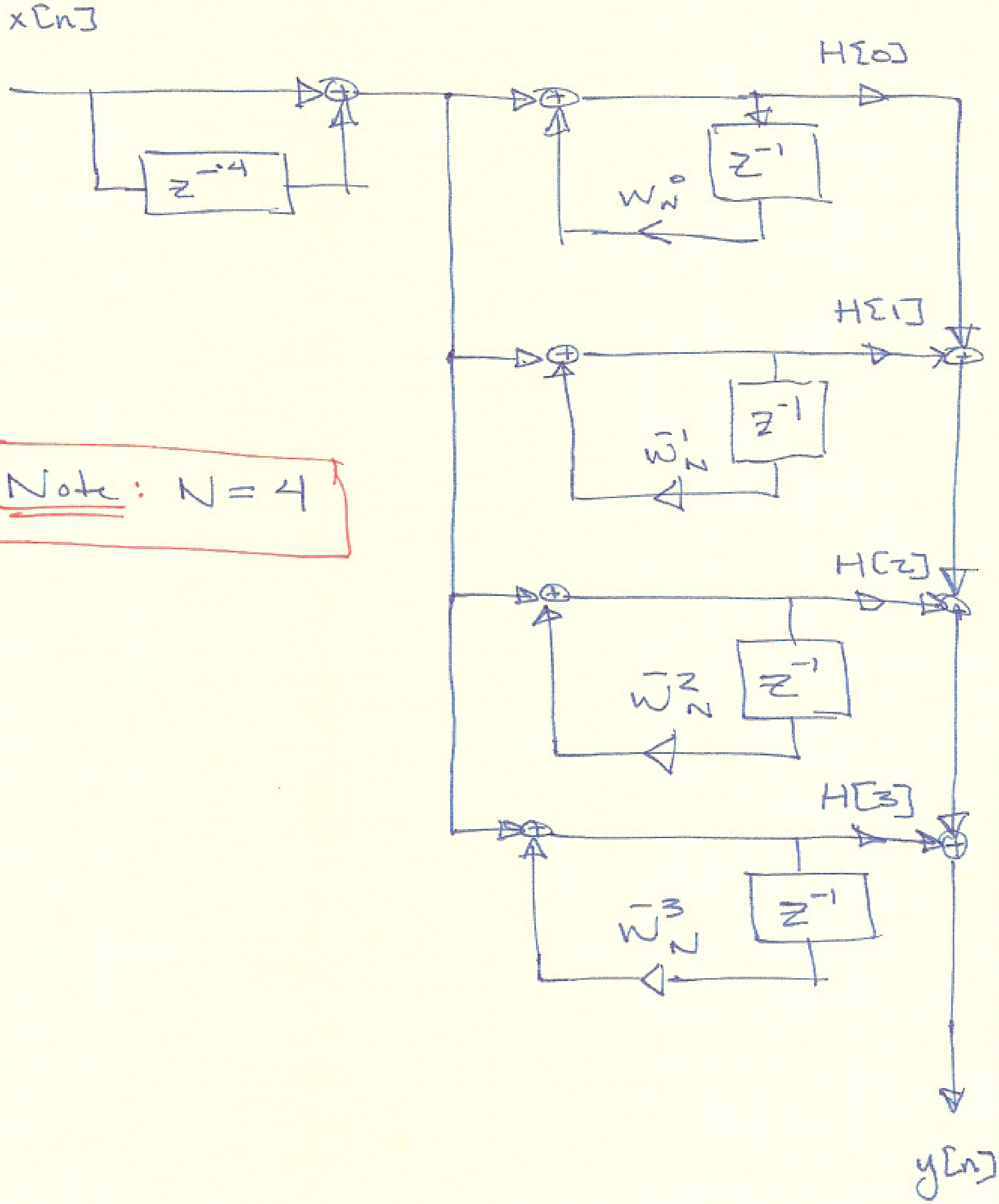
$$\boxed{H(z) = \frac{1 - z^{-N}}{N} \sum_{k=0}^{N-1} \frac{H[k]}{1 - W_N^{-k} z^{-1}}} \quad (1)$$

Note: $\boxed{W_N^{-kN} = 1}$

If it is shown that (1) has the responce form similiar to IIR filter. However the resulting filter is an FIR as the poles will be cancelled out by the roots of $1 - z^{-N} = 0$

Based on the system function (1) a parrellel structure can be drawn for N=4 as follows.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-4}}{4} \sum_{k=0}^{3} \frac{H[k]}{1 - W_4^{-k} z^{-1}}$$



Note: $N = 4$

EX: Determine the freq response of a filter with $h[n] = \{1, 1, 1\}$

SOL:

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]\, e^{-j\omega n}$$

$$= 1 + e^{-j\omega} + e^{-2j\omega}$$

$$= e^{-j\omega}\left(e^{j\omega} + 1 + e^{-j\omega}\right)$$

$$= (1 + 2\cos\omega)\, e^{-j\omega}$$

$$= \left|H(e^{j\omega})\right| \angle H(e^{j\omega})$$

$$\left|H(e^{j\omega})\right| = \left|1 + 2\cos\omega\right|$$

$$\angle H(e^{j\omega}) = \begin{cases} -\omega & 0 \le \omega \le \frac{2\pi}{3} \\ \pi - \omega & \frac{2\pi}{3} \le \omega \le \pi \end{cases}$$

The phase response is a peicewise linear function and hence, discontinoous.

For applitude response,

$$H(e^{j\omega}) = H_r(\omega) \angle H(e^{j\omega})$$

amplitude response.

The phase response associated with amplitude response is a continuous function of ω.

Linear Phase filter

There are 4 types of linear phase filters. To study these filters we will write:

$$H(e^{j\omega}) = H_r(\omega)\, e^{j(\beta - \alpha\omega)}$$

$$\beta = 0, \pm\frac{\pi}{2}$$

$$\alpha = \frac{N-1}{2}$$

* Type I ($h[n]$ symmetric, N-odd)

$$h[n] = h[n-N-1]$$

$$\therefore \quad H(e^{j\omega}) = \sum_{n=0}^{N-1} h[n]\, e^{-j\omega n}$$

$$= \left\{ h[0] + h[N-1]\, e^{-j\omega(N-1)} \right\}$$

$$+ \left\{ h[1]\, e^{-j\omega} + h[N-2]\, e^{-j\omega(N-2)} \right\}$$

$$+ \ldots$$

Consider the second term.

$$h[1]\left\{ e^{-j\omega} + e^{-j\omega(N-2)} \right\}$$

$$h[1]\, e^{-j\omega\left(\frac{N-1}{2}\right)} \left\{ e^{j\omega\left(\frac{N-1}{2} - 1\right)} + e^{j\omega\left(\frac{N-1}{2} - N+2\right)} \right\}$$

$$h[1]\, e^{-j\omega\left(\frac{N-1}{2}\right)} \left\{ e^{j\omega\left(\frac{N-1}{2} - 1\right)} + e^{j\omega\left(\frac{N-1}{2} - 1\right)} \right\}$$

$$h[1]\, e^{-j\omega\left(\frac{N-1}{2}\right)} \cos\left(\omega\left(\frac{N-1}{2} - 1\right)\right)\cdot(2)$$

$$\therefore \quad H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{i=0}^{\frac{N-1}{2}} (2)\, h[i] \cdot \cos\left(\omega\left(\frac{N-1}{2} - i\right)\right)$$

$$= H_r(\omega)\, e^{-j\omega\left(\frac{N-1}{2}\right)}$$

$$\therefore \quad \omega = 0, \pi \longrightarrow H_r(\omega) \neq 0$$

$$\omega = \pi/2 \longrightarrow H_r(\omega) \text{ may be } 0.$$

Hence this filter is good for LP & HP. But may not be good for BP.

* Type-2 (symmetric, N-even)

This is very similiar to the last derivation.